1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 6,Module 5, Numerical problems based on Gravitation Chapter 8, Gravitation
Module Id	Keph_10805_eContent
Pre-requisites	Gravitational force, acceleration due to gravity, Kepler's laws, Gravitational potential energy, kinetic energy, orbital velocity and escape velocity.
Objectives	After going through this lesson, the learners will be able to: • **Solve** different types of numerical problems based on Gravitation • **Analys**e a problem and relate it to the concept involved in the problem
Keywords	Principle of superposition, variation of g with height and depth, Kepler's laws , Satellites, orbital velocity, geostationary satellites, Mongolian, International space station, INSAT satellite

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd. Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	SmitaFangaria	PGT Physics Developer Anvishika Amity International School, Noida
Review Team	Associate Prof. N.K. Sehgal (Retd.) Prof. V. B. Bhatia (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

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1. UNIT SYLLABUS

Unit VI: Gravitation

Chapter 8: Gravitation

Kepler's laws of planetary motion, universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential; escape velocity; orbital velocity of a satellite; Geo-stationary satellites.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 5 Modules

The above unit is divided into five modules for better understanding.

Module 1	Gravitation Laws of gravitation Early studies Kepler's laws
Module 2	Acceleration due to gravity Variation of g with altitude Variation of g due to depth Other factors that change g
Module 3	Gravitational field Gravitational energy

	Gravitational potential energy Need to describe these values
Module 4	Satellites India's satellite programme and target applications Geo stationary satellites and Polar satellites Escape velocity India's space program
Module 5	Numerical problems based on Gravitation

Module 5

3. WORDS YOU MUST KNOW

Solar system: The sun, its eight planets with their satellites and the asteroid belt constitutes the solar system.

Vectors: Physical quantities which have both magnitude and direction. They are added according to the vector laws of addition.

Linear momentum: **(P=** m **v)** A physical quantity which is the product of mass and velocity. The rate of change of linear momentum is equal to the external force.

$$(F = \frac{dP}{dt})$$

Law of conservation of linear momentum: According to this law, the linear momentum of the system remains constant if the external force is zero**.**

Angular momentum: (L = m v r when v is perpendicular to r). It is the analogue of linear momentum in rotational motion. The rate of change of angular momentum is torque:

$$\tau = \frac{dL}{dt}.$$

Law of conservation of angular momentum: According to this law, the angular momentum of the system remains constant if the external torque is zero.

Gravitational force: Force of attraction between two masses which is proportional to the product of the masses and inversely proportional to the square of the distance between them.

Gravitational potential energy: Potential energy by virtue of the gravitational force.

Kinetic energy: Energy possessed by an object by virtue of its motion**.**

Mechanical energy: The sum of kinetic and potential energy.

Projectile: An object moving only under the influence of gravity.

Centripetal force: Radial force towards the centre of the circular path in which the object is moving.

Keplar's laws of planetary motion

Law of orbits : All planets move in elliptical its with the Sun situated at one of the foci of the ellipse

Law of areas : The line that joins any planet to the sun sweeps equal areas in equal intervals of time

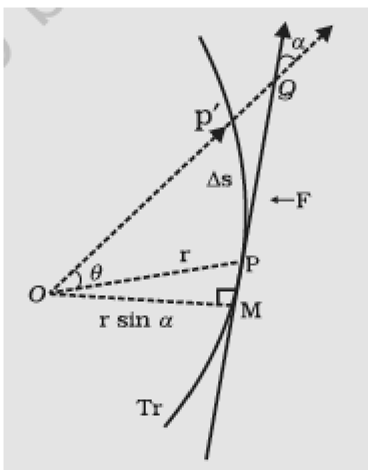
Law of periods : The square of the time period of revolution of a planet is proportional to the
cube of the semi-major axis of the ellipse traced out by the planet.

Principle of superposition If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses

4. INTRODUCTION

In the earlier modules of this unit we considered the force of mutual attraction between masses.

This force is universal and acts between all masses irrespective of its state , shape and composition It is a central force The angular momentum of the particle is conserved, if the torque _ = r × F due to the force F on it vanishes. This happens either when F is zero or when F is along r. We are interested in forces which satisfy the latter condition. Central forces satisfy this condition. A 'central' force is always directed towards or away from a fixed point, i.e., along the position vector of the point of application of the force with respect to the fixed point.



Further, the magnitude of a central force *F* depends on *r*, the distance of the point of application of the force from the fixed point; *F* = *F*(*r*).
In the motion under a central force the angular momentum is always conserved. Two important results follow from this:
(1) The motion of a particle under the central force is always confined to a plane.
(2) The position vector of the particle with respect to the centre of the force (i.e. the fixed point) has a constant areal velocity. In other words the position vector sweeps out equal areas in equal times as the particle moves under the influence of the central force.
The gravitational force of the sun on the planet is directed towards the sun.
This force also satisfies the requirement *F = F(r)*,
since

$$F = \frac{GM_1M_2}{r^2}$$

where*m*1 and *m*2 are respectively the masses of the planet and the sun and *G* is the universal constant of gravitation.

We have learnt the principle of superposition .we will use these ideas to solve some problems. These will ensure greater clarity.

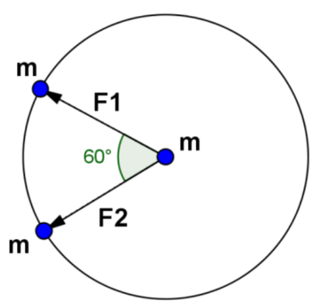
We will do some examples under the following categories

- **Numerical based on Principle of superposition.**
- **Numerical based on the variation of g with depth and height.**
- **Numerical based on Kepler's laws.**
- **Numerical based on the orbital velocity, time period and energy of natural and artificial satellite.**

5. NUMERICAL BASED ON THE PRINCIPLE OF SUPERPOSITION

EXAMPLE 1:

Three identical point masses each of 1kg lie in the arrangement as shown below. The radius of the circle is 1m. What is the magnitude of the gravitational force on the mass placed at the centre due to the other two masses? Here G is the universal gravitational constant.



SOLUTION:

According to this principle, if we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses. Hence the force on the central mass is:

$$\vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2}$$

$|F_1| = |F_2| = \frac{Gmm}{R^2} = \text{G}$ (as m = 1kg and R = 1m)

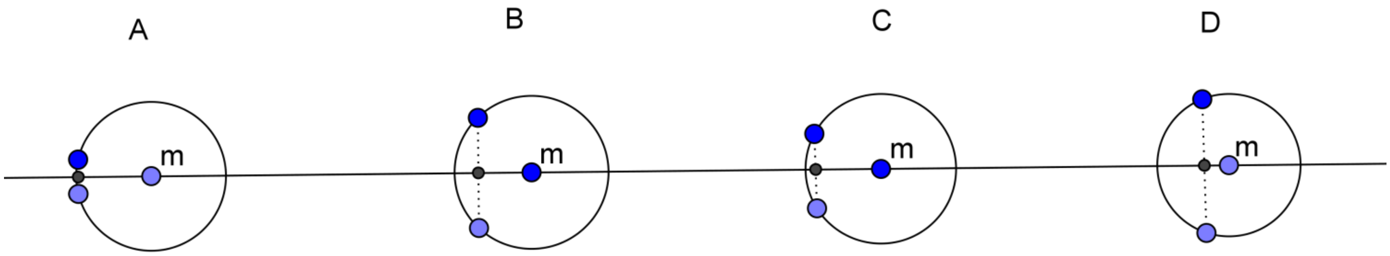
Magnitude of force on the central mass:

$$= \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2cos60}$$

$$= \sqrt{G^2 + G^2 + 2GGcos60} = \mathbf{G\sqrt{3}N}$$

EXAMPLE 2:

The figure below shows four circles of same radius at the centre of which lies a point mass m. Two other identical point masses lie on each circle. The centre of mass of these two point masses is also shown in each case on the central axis.



In which of the cases:

Gravitational force on central mass m is maximum.

Ans: A

Gravitational force on central mass m is minimum.

Ans: D

Explanation:

Magnitude of force on the central mass in all the given cases above

$$= \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2cos\theta}$$

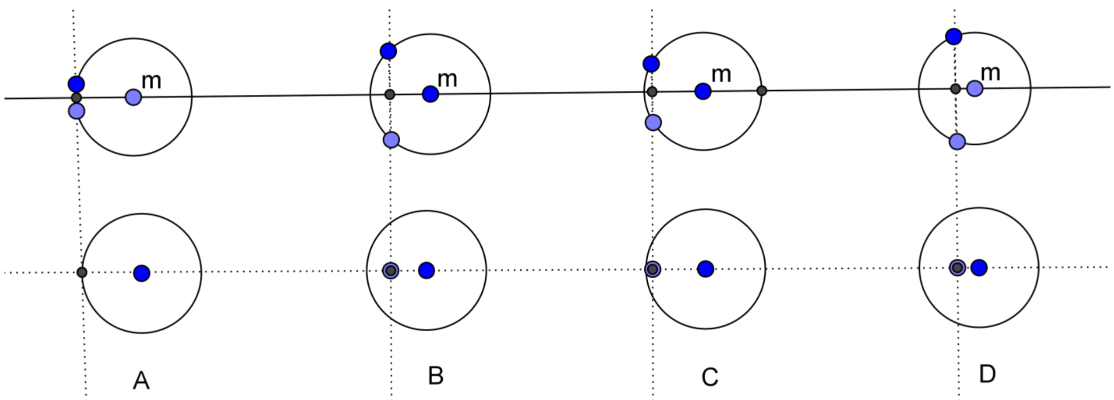
Also in all the cases $|F_1| = |F_2|$

The masses are identical and are at a distance equal to the radius of the circle.

Hence the force on the central mass only depends on the cosine of the angle between the forces F_1 and F_2. Lesser the angle, greater is the force on the central mass and vice versa.

EXAMPLE 3:

Now in the above situation the two identical masses on the circle are removed and replaced by a point mass whose position and mass exactly matches the centre of mass of the removed particles.



In this new situation in which of the two cases:

Gravitational force on the central mass is maximum.

Ans: D

Gravitational force on the central mass is minimum.

Ans: A

EXPLANATION :

We see that if two masses are replaced by their centre of mass the results are very different. Hence in the superposition principle we need to consider the force between each individual pair. We cannot replace the rest of the masses by their centre of mass.

EXAMPLE 4:

Two particles of mass m and 4m are fixed on an axis. Where on the axis a third particle of mass 2m is placed (other than infinity) so that the net gravitational force on it zero.

A. **Between the two particle of masses m and 4m and closer to the particle of 4m mass.**
B. **Between the two masses m and 4m and closer to the particle of m mass.**
C. **Left of particle of mass m**
D. **Right of particle of mass 4m.**

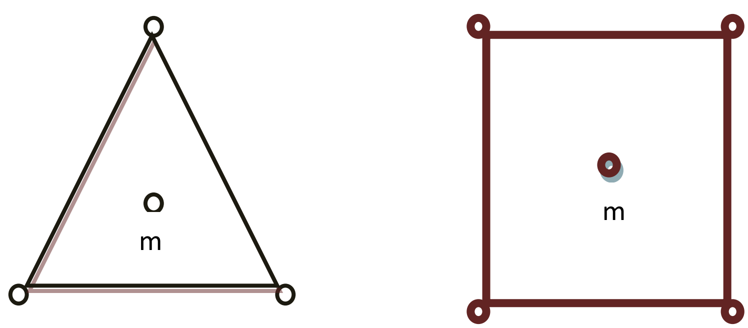
Ans: B

SOLUTION:

The gravitational force of two masses can be cancelled only on the line joining them between the two masses. This point where the force is cancelled is called the point of equilibrium. It will always lie nearer the smaller mass.

EXAMPLE 5:

Figure shows identical point masses kept on the vertices of a equilateral triangle and square. What is the force experienced by a particle of mass m kept at the geometrical centre of the figures?



SOLUTION:

The gravitational force at the geometrical centre of geometrical polygons is always zero if identical masses are kept at their vertices. The net force at the centre is due to the vector sum of all the forces it experiences due to the masses on the vertex of the polygon. The vector sum of all these forces is zero at the centre.

6. NUMERICAL BASED ON THE VARIATION OF ACCELERATION DUE TO GRAVITY(g) WITH DEPTH AND HEIGHT

EXAMPLE 6:

What will be the new value of g experienced by an object which is taken from the surface of the earth to heights equal to?

R
2R
3R

From the above result find the new value of g at a height of nR from the surface of the earth. Here n= natural number (0,1,2,3,4,5____)

SOLUTION:

$g^{/}= \frac{GM}{(R+h)^2}$ and $g= \frac{GM}{R^2}$

At h = R, $g^{/} = \frac{GM}{(R+R)^2} = g/4$

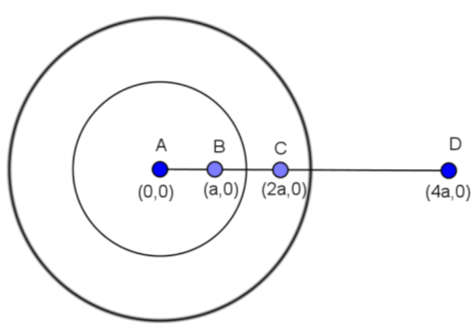
At h =2R, $g^{/} = \frac{GM}{(R+2R)^2} = g/9$

At h =3R, $g^{/} = \frac{GM}{(R+3R)^2} = g/16$

At h =nR, $g^{/} = \frac{GM}{(R+nR)^2} = g/(n+1)^2$

EXAMPLE 7:

Two concentric shells of identical mass M are as shown in the adjacent figure. What is the gravitational force experienced by point objects each of mass m placed at the points A, B, C and D. Co-ordinates of the particles are shown.



SOLUTION:

This question is based on the shell theorem

Force on objects at A and B is zero as they are inside both the shells.

Force on object C is due to the inner shell of mass M which behaves as if its mass is concentrated at the centre.

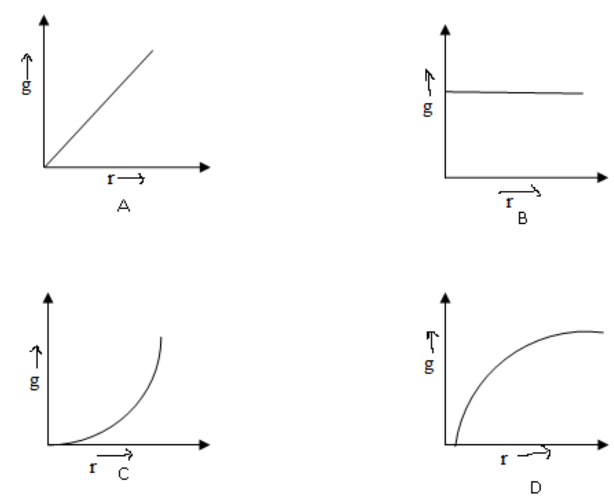
Hence, F(at C) = $\frac{GMm}{4a^2}$

Force on object D is due to both the shells which behave as their entire mass 2M is concentrated at the centre.

Hence, F(at D) = $\frac{GMm\times2}{16a^2} = \frac{GmM}{8\,a^2}$

EXAMPLE 8:

Which of the graph shown below represents the variation of g with distance r form the centre O of the earth to its surface?



ANSWER:A.

This is because the value of g increases linearly with distance from the centre of the earth. It reaches its maximum value at the surface of the earth.

EXAMPLE 9:

If an astronaut whose height is h is 1.7m is floating with his feet down in an orbiting space shuttle at a distance of 6.77 x 10^6m from the center of a black hole of mass, what will be the difference between the acceleration due to gravity of the black hole at her feet and her head?

SOLUTION:

The height of the astronaut is very small in comparison to the orbital radius, so the small change in g can be found by differentiating the equation:

g = $\frac{GM}{r^2}$

dg = -2$\frac{GM}{r^3}$dr

Substituting, r = 6.77 x 10^6m and dr = 1.7m
= - 14.5m/s^2

So, there is a substantial difference in the value of g at the astronaut's feet and head which will be very painful and will cause his body to stretch.

EXAMPLE 10

A 100gram apple is falling towards the earth from a height of 10m from the surface of the earth. Find the acceleration with which the earth moves towards the apple. (Mass of the earth = 6 x 10^{24}kg).

SOLUTION:

The distance between the center of the earth and the apple is the sum of the radius of the earth and the height of 10m which is approximately equal to the radius of the earth = 6.37 x 10^6m. Hence the acceleration due to gravity g for the apple is almost equal to 9.8 m/s^2.

Mass of the earth M= 6 x 10^{24}kg, mass of apple = 100g = 0.1 kg

Ma = mg

a = $\frac{mg}{M}$ = (0.1 x 9.8) / 6 x 10^{24} = 1.63 x 10^{-25}m/s^2.

CONCLUSION:

The acceleration of the earth towards the apple is negligibly small. This is the reason why we always see objects falling towards the earth and not the earth rising to meet them. The earth appears to be stationary.

EXAMPLE 11:

The value of acceleration due to gravity on the surface of moon is one sixth of that on earth. A balloon filled with hydrogen will:

Fall with acceleration g on the moon
Rise with acceleration g on the moon
Fall with acceleration g/6 on the moon
Rise with acceleration g/6 on the moon

SOLUTION:C

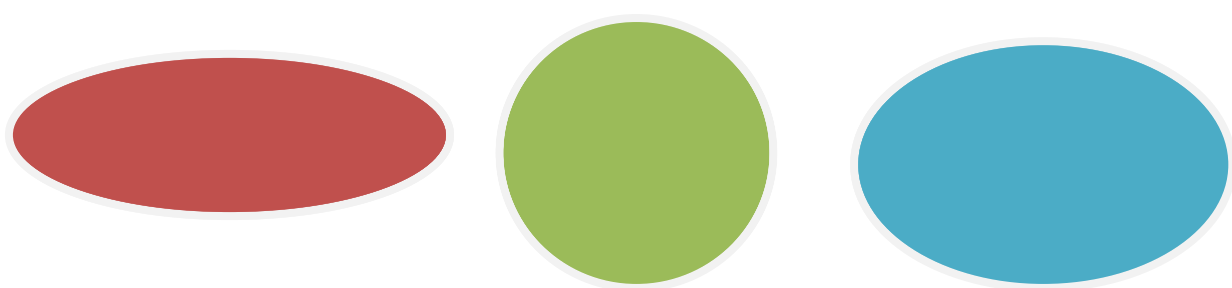
The acceleration of the balloon will be g/6 in the downward direction. On earth we have atmosphere which causes a buoyant force on the balloon due to the volume of the air displaced by it. This buoyant force acts in the upward direction and is equal to weight of the air displaced by it. If the weight of the air filled in the balloon is less than this buoyant force the balloon will rise up.

On the moon there is no atmosphere due to its less escape velocity, so the balloon will experience no buoyant force and it will fall down with an acceleration of g/6.

7. NUMERICAL BASED ON KEPLER'S LAWS

EXAMPLE 12:

Rank the given figures according to the decreasing eccentricity of the ellipse greatest first



(a) (b) (c)

SOLUTION:

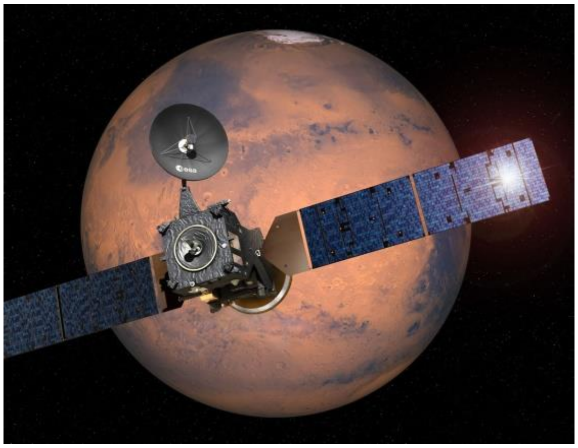
a, c, b

The ovalness of an ellipse decides the eccentricity of the ellipse. The more it is oval greater will be its eccentricity. Eccentricity of an ellipse varies between 0 and 1. Circle is a special ellipse with eccentricity 0.

EXAMPLE 13:

The orbit of Mangalyaan (Mars Orbiter mission) sent by ISRO is highly eccentric. It is at a distance of 420km when it is closest from the surface of the planet Mars and at a distance of 72000 km from the surface when it is farthest from Mars? (Radius of mars = 3390km). What is ratio of its velocity at perihelion and aphelion?

SOLUTION:



This is based on the Kepler's law of areas.

The angular momentum at perihelion will be equal to the angular momentum at aphelion.

Perihelion (r_1) = 3390 + 420 = 4410 km

Aphelion (r_2) = 3390 + 72000 = 75390 km

$$mv_1r_1 = mv_2r_2$$

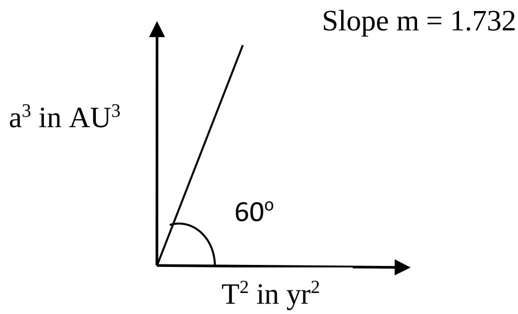
$$\text{Hence} \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

$$= \frac{75390}{4410} = \mathbf{17}$$

Hence, the velocity of Mangalyaan is more at perihelion**17 timesthat of aphelion.**

EXAMPLE 14:

The following graph with slope m is shown for the cube of average orbital radius of planets (a^3 in AU^3) and the square of the time period (T^2 in yr^2). Spot the mistake in the graph.



SOLUTION:

Slope m should be unity. So the line should make an angle of 45^o instead of 60^o.

This is based on Kepler's law of periods which states that the square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

In equation form this is $T^2 \propto r^3$

Hence, $T^2/r^3 = \text{constant}$

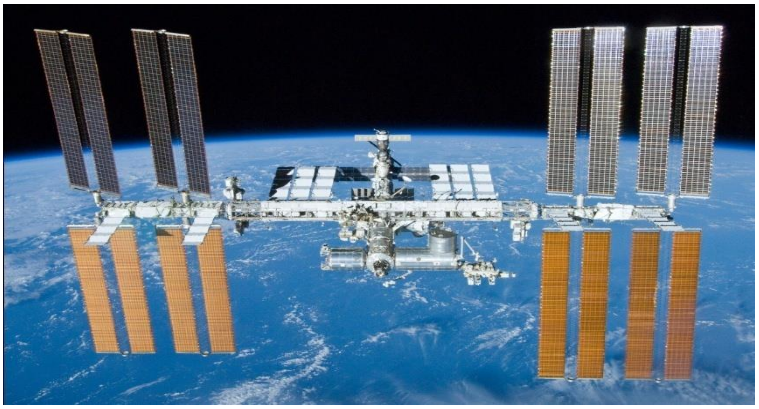
If T is measured in earth years and r in the astronomical units, this constant is unity.

Hence, $T^2 = r^3$

8. NUMERICAL BASED ON THE ORBITAL VELOCITY, TIME-PERIOD AND ENERGY OF NATURAL AND ARTIFICIAL SATELLITE

EXAMPLE 15:

What is total mechanical energy possessed by a 80kg astronaut who is inside the International space station which is orbiting the earth at a height of 400km from the surface of the earth. Take the radius of earth = 6400 km and mass of the earth to be M = 5.98 x 10^{24} kg. Also find the orbital velocity of the International space station.



SOLUTION:

Total energy possessed by the astronaut of mass 80 kg

$$= -\frac{GMm}{2r}$$

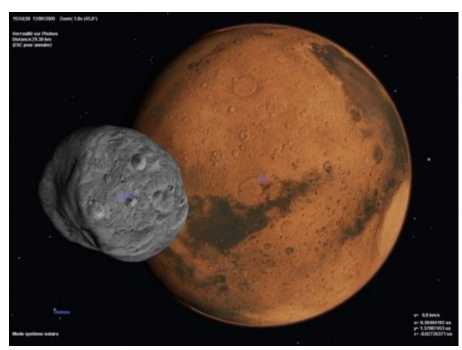
$$= -\frac{6.67\times10^{-11}\times 80\times5.98\times10^{24}}{2\times6.8\times10^{6}}$$

$$= -2.35 \times 10^{9}\,\text{J}$$

Orbital velocity of International space station = $\sqrt{\frac{GM}{r}}$ = 7.66 km/s

EXAMPLE 16:

Phobos is a moon of the planet Mars .It orbits around Mars with an orbital radius of 9380 km. Its orbital period is 0.319 days (2.77× 10^4 seconds). Determine the mass of Mars based on this data.



SOLUTION:

Using Kepler's law of periods and the data given for the time period and orbital radius of Phobos.

$T = 2.77 \times 10^{4}$ seconds

and r = 9380 km

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Mass of mars = $M = \frac{4\pi^2 r^3}{GT^2}$ = 6.39 × 10^{23} kg

EXAMPLE 17:

GSAT-11 is a geostationary Indian communication satellites made by ISRO which isproposed to be launched above the equator for orbiting the earth and make one complete orbit every 24 hours. Because the orbital period is synchronized with the Earth's rotational period, a geostationary satellite can always be found in the same position in the sky relative to an observer on Earth.

(GIVEN: M_{Earth} = 5.98 x 10^{24} kg)

a. Determine its orbital radius.
b. Determine its orbital speed and escape speed.
c. Determine the centripetal acceleration

SOLUTION:

$r^3 = \frac{GMT^2}{4\pi^2}$

From the given data the orbital radius of the geostationary satelliter =42,164 km

Orbital speed of the satellite $v = \sqrt{\frac{GM}{r}}$ = 3.07 km/s

Escape speed = $\sqrt{2}$ x orbital speed = 4.34 km/s

Centripetal acceleration of the geostationary satellite = v^2/r = 0.223 m/s^2

These geostationary satellites are very useful for communication purposes.

9. SUMMARY

In this module we have learnt to solve:

- Numerical based on Principle of superposition.
- Numerical based on the variation of g with depth and height.
- Numerical based on Kepler's laws.
- Numerical based on the orbital velocity, time period and energy of natural and artificial satellite.